A Mechanism for the Top-Bottom Mass Hierarchy

MICHIO HASHIMOTO [1], and SHINYA KANEMURA [2]
[1] *Department of Physics, Pusan National University, Pusan 609-735, Korea*
[2] *Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan*
[1] michioh@charm.phys.pusan.ac.kr
[2] kanemu@het.phys.sci.osaka-u.ac.jp

ABSTRACT

We discuss a mechanism to generate hierarchy between masses of the top and bottom quarks without fine-tuning of the Yukawa coupling constants. In the framework of the two-Higgs-doublet model (THDM) with a discrete Z_2 symmetry, there exists the vacuum where only the top quark receives the mass of the order of the electroweak symmetry breaking scale $v(\simeq 246\,\mathrm{GeV})$, while the bottom quark remains massless. We show a model in which a small sofb-breaking mass $m_3^2[\sim v^2/(4\pi)^2]$ for the Z_2 symmetry is generated by the dynamics above the cutoff scale of the THDM. The appearance of m_3^2 gives the small mass for the bottom quark, and explain the hierarchy $m_b/m_t \ll 1$.

1. Introduction

The measured quark mass spectrum shows a specific feature. Only the top quark has the mass of the order of the electroweak symmetry breaking (EWSB) scale v ($= (\sqrt{2}G_F)^{-1/2} \simeq 246$ GeV), while masses of the other quarks are much smaller. In the Standard Model (SM), the unique Higgs doublet field $\Phi_{\rm SM}$ is responsible for the EWSB and gives masses of all quarks via the Yukawa interactions; i.e., $m_f \simeq y_f\langle\Phi_{\rm SM}\rangle$ with $\langle\Phi_{\rm SM}\rangle = (0, v/\sqrt{2})^T$. The observed mass spectrum is obtained only by assuming unnatural hierarchy among the Yukawa coupling constants y_f. Nevertheless, no explanation is given for such fine tuning in the SM.

In this Talk, we present an alternative scenario in which the quark mass spectrum is reproduced without fine tuning in magnitude of the Yukawa coupling constants[1]. We study the hierarchy between m_t and m_b under the assumption of $y_t \sim y_b \sim \mathcal{O}(1)$. In order to realize $m_b/m_t \sim 1/40$ in a natural way, we consider the two-Higgs-doublet model (THDM) with Φ_1 and Φ_2, imposing the discrete Z_2 symmetry[2], in which only Φ_1 couples to the bottom quark while Φ_2 does to the top quark. The hierarchy $m_t \gg m_b$ is then equivalent to $v_2 \gg v_1$, where $\langle\Phi_{1,2}\rangle = (0, v_{1,2}/\sqrt{2})^T$. When the Z_2 symmetry is exact, there exists the vacuum with $v_1 = 0$ and $v_2 = v$. A nonzero value of v_1 ($\ll v_2$) is induced as a perturbation of a small soft-breaking parameter m_3^2 for the Z_2 symmetry. The small $m_3^2[\sim v^2/(4\pi)^2]$ is generated by the dynamics above the cutoff scale of the THDM. Consequently, we obtain $m_b/m_t \ll 1$. This scenario can be extended to include the first two generation quarks.

2. Minimal Model

We first consider only the top and bottom quarks among fermions, and discuss the extension for the other quarks later on. Details are shown in Ref. [1]. In order to describe

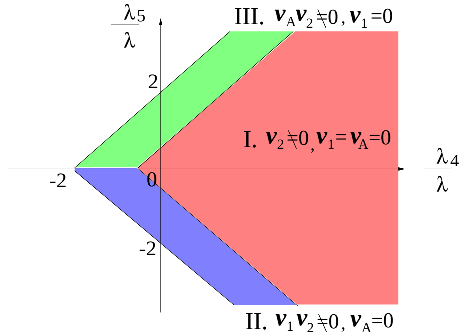

Figure 1: Vacuum structure for $m_3^2 = 0$ and $m_2^2 < -|m_1^2|$.

the assumption of $y_t \simeq y_b$, we introduce the global $SU(2)_R$ symmetry[3,4], in addition to the $SU(2)_L$ gauge symmetry:

$$q_{L,R} \to q'_{L,R} = U_{L,R}\, q_{L,R}\,, M_{21} \to M'_{21} = U_L M_{21} U_R^\dagger, \tag{1}$$

where $q_{L,R} \equiv (t_{L,R}, b_{L,R})$ and $U_{L,R} \in SU(2)_{L,R}$, respectively. The 2×2 matrix M_{21} is defined by $M_{21} \equiv \left(\tilde{\Phi}_2, \Phi_1\right)$, with $\tilde{\Phi}_2 = i\tau_2 \Phi_2^*$. The Z_2 symmetry can be expressed in terms of $q_{L,R}$ and M_{21} by $q_L \to q'_L = q_L$, $q_R \to q'_R = \tau_3 q_R, M_{21} \to M'_{21} = M_{21}\tau_3$. The Yukawa interaction then is written as $\mathcal{L}_Y = -y\bar{q}_L M_{21} q_R + (\text{h.c.})$, with $y \equiv y_t = y_b$. The Higgs potential with the softly-broken Z_2 symmetry is given by

$$\begin{aligned} V(M_{21}) &= \frac{1}{2}m^2 \mathrm{tr}(M_{21}^\dagger M_{21}) - \frac{1}{2}\Delta_{12}\mathrm{tr}(M_{21}^\dagger M_{21}\tau_3) - \left[m_3^2 \det M_{21} + (\text{h.c.})\right] \\ &+\lambda\left[\mathrm{tr}(M_{21}^\dagger M_{21})\right]^2 + 2\lambda_4 \det(M_{21}^\dagger M_{21}) + \left[\lambda_5(\det M_{21})^2 + (\text{h.c.})\right]. \end{aligned}$$

The Z_2 symmetry is softly broken by the mass term of m_3^2. A non-zero value of Δ_{12} measures the soft breaking of the global $SU(2)_R$ symmetry. In order to evade explicit CP violation, we choose the phases in m_3^2 and λ_5 to be zero.

Let us consider the effective potential $V(\langle M_{21}\rangle)$ to study the vacuum structure. By using $SU(2)_L$ and $U(1)_Y$, the VEV's in the THDM can be generally parameterized as

$$\langle M_{21}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v_2 & v_E \\ 0 & v_1 + iv_A \end{pmatrix}. \tag{2}$$

In our model, it is easily shown that $v_E = 0$ at the tree level. Since spontaneous CP violation does not occur for $m_3^2 = 0$, three types of the nontrivial vacuum are possible at the tree level: (a) $v_1 = v_A = 0, v_2 \neq 0$, (b) $v_1 v_2 \neq 0, v_A = 0$, (c) $v_A v_2 \neq 0, v_1 = 0$. In Fig. 1, the area (I) corresponds to the vacuum (a), while the areas (II) and (III) do to the vacua (b) and (c), respectively. In order to realize $m_b/m_t \ll 1$ without fine tuning, we choose the vacuum (a) which leads to

$$m_t = \frac{1}{\sqrt{2}}\, y\, v, \quad m_b = 0. \tag{3}$$

We now switch on a *small* soft-breaking parameter $m_3^2(\ll v^2)$ of the discrete Z_2 symmetry. We do not consider the possibility of spontaneous CP violation. A nonzero v_1 is

necessarily induced for $m_3^2 \neq 0$ from the stationary condition. As a perturbation from the vacuum (a) with $m_3^2 = 0$, we consequently obtain

$$\frac{v_1}{v_2}(\equiv \frac{1}{\tan\beta}) = \frac{m_3^2}{m_H^2}\left\{1 + \mathcal{O}\left(\frac{m_3^4}{v^4}\right)\right\}. \tag{4}$$

The masses of the top and bottom quarks are given by

$$m_t \simeq \frac{1}{\sqrt{2}}\, y\, v, \quad m_b = \frac{1}{\sqrt{2}}\, y\, v_1, \tag{5}$$

so that the bottom quark finally obtain the small mass. The mass hierarchy of m_t and m_b then is deduced from Eqs. (4) and (5) without fine tuning of the Yukawa coupling constants; i.e., $m_t/m_b = \tan\beta$. With nonzero m_3^2 the Higgs doublets Φ_1 and Φ_2 do mix. The mixing angle $\beta - \alpha$ is expressed as

$$\sin(\beta-\alpha) = 1 - \left(\frac{m_H^2 - m_{H^\pm}^2}{m_H^2 - m_h^2}\right)^2 \frac{2}{\tan^2\beta} + \mathcal{O}\left(\frac{m_3^6}{v^6}\right), \tag{6}$$

From Eq. (6), the property of the CP-even Higgs boson h is very similar to the SM one. The masses of the extra Higgs bosons H, A and $H^\pm$ turn out to be of the order of v.

We now give comments on the case with relaxed $SU(2)_R$. First, it can be shown that Eq. (4) does not change. Second, although Eq. (6) is slightly modified, the essential result of $\sin(\beta-\alpha) = 1 - \mathcal{O}(\tan^{-2}\beta)$ still holds. Also, the masses of the extra Higgs bosons remain to be of $\mathcal{O}(v)$.

3. A mechanism for small m_3^2.

Let us consider a model with a complex scalar field S which is a $SU(2)_L$ singlet without $U(1)_Y$ charge: $\mathcal{L} = \mathcal{L}_{\rm kin} - V_\Phi - V_S - V_{\not Z_2}$, where $\mathcal{L}_{\rm kin}$ represents the kinetic term and V_Φ is the Z_2 symmetric part of the THDM potential with $m_3^2 = 0$. The potential V_S for the complex scalar S and the interaction term $V_{\not Z_2}$ between S and $\Phi_{1,2}$ are given by $V_S = M_S^2 S^\dagger S + \kappa (S^\dagger S)^2 + V_{Z_{2n}}$, $M_S^2 > 0$, with

$$V_{Z_{2n}} = \frac{\eta}{\Lambda^{2n-4}}\left(S^{2n} + \text{h.c.}\right), \quad \eta \sim \mathcal{O}(1), \tag{7}$$

and

$$V_{\not Z_2} = \frac{\xi}{\Lambda^{2\ell-2}}\left(S^{2\ell}\Phi_1^\dagger\Phi_2 + \text{h.c.}\right), \quad \xi \sim \mathcal{O}(1), \tag{8}$$

respectively. In Eqs. (7) and (8), Λ denotes the cutoff scale of the model. We now set $n = 1$ (case A) or $n = \ell$ (case B) with $\ell \geq 1$. We note that V_S has the Z_{2n} symmetry under $S \to e^{i\frac{\pi}{n}}S$, while V_Φ is Z_2 invariant under the transformation $\Phi_1 \to -\Phi_1$, $\Phi_2 \to +\Phi_2$. The interaction term (8) explicitly breaks both Z_{2n} and Z_2. Some invariant terms under Z_{2n} and Z_2 are not explicitly included here, as they are irrelevant to our conclusion.

Supposing that $M_S(\sim \Lambda)$ is much larger than the EWSB scale, we integrate out the field S and thereby obtain the THDM with the softly-broken Z_2 symmetry ($m_3^2 \neq 0$) as the low-energy effective theory. We estimate

$$m_3^2 \sim \xi\eta^\ell \frac{1}{(4\pi)^{2\ell}} M_S^2, \qquad \text{for Case A}, \tag{9}$$

$$m_3^2 \sim \xi\eta \frac{1}{(4\pi)^{2(2\ell-1)}} M_S^2, \qquad \text{for Case B}. \tag{10}$$

If we take the cutoff $M_S = 4\pi v$ for Case A or $M_S = (4\pi)^2 v$ for Case B, we can obtain $m_3^2 \sim v^2/(4\pi)^2$ for $\ell = 2$.

4. Quark Mass Matrices

Let us consider the extension of our model incorporating first two generation quarks. Under the discrete symmetry [2], two types of Yukawa interactions are possible in the THDM, so called Model I and Model II[5]. Obviously Model I is inconsistent with our scenario, so that we here apply Model II. We then assume that 3×3 matrices Y_U^{ij} and Y_D^{ij} of the Yukawa coupling with up- and down-type quarks take the following forms, $Y_U^{ij} \sim Y_D^{ij} \sim y, \forall(i,j), \quad y \sim \mathcal{O}(1)$, which lead to $m_t \gg m_c, m_u$ and $m_b \gg m_s, m_d$, and the KM matrix becomes approximately diagonal. We can numerically reproduce the data for the mass spectrum and the KM matrix, allowing small fluctuations of the Yukawa coupling constants. Although we can avoid hierarchy among Yukawa couplings in this way, subtle cancellation among the $\mathcal{O}(1)$ mass-matrix elements is required to obtain masses of light quarks.

It is possible to apply our scenario to the lepton sector. The tau lepton then receives the small mass due to the similar mechanism to the bottom quark. In this case, the Dirac mass of the tau neutrino could be produced around m_t. To explain the tiny (Majorana) mass of the tau neutrino, additional mechanism such as the seesaw might be helpful.

5. Summary and Discussions

We have proposed the mechanism to explain the mass hierarchy between the top and bottom quarks without fine tuning, starting from the vacuum with $(v_1, v_2) = (0, v)$. Such a vacuum can exist when the Z_2 symmetry is exact. The observed mass spectrum $m_t \gg m_b \neq 0$ is realized via the small soft-breaking parameter m_3^2 for the Z_2 symmetry. We have presented the model in which a small m_3^2 is induced from the underlying physics above the cutoff scale of the THDM. The size of $\tan\beta$ corresponds to the ratio $m_t/m_b \sim 40$. The masses of the extra Higgs bosons H, A and $H^\pm$ are expected to be $\mathcal{O}(v)$. In addition to the SM-like Higgs boson h, all the extra Higgs bosons in our model are expected to be discovered at the CERN Large Hadron Collider (LHC). Our prediction of $\sin(\beta - \alpha) \simeq 1$ can also be confirmed at the LHC and Linear Collider's (LC's). Our scenario may further be tested by measuring the hhh coupling at future LC's[6].

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